A Framework for Inertial Sensor Calibration Using Complex Stochastic Error Models

Stebler Y.*, Guerrier S.**, Skaloud J.*, and Victoria-Feser M.-P.**

* Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland
** University of Geneva, Switzerland

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Errors in Inertial Sensors

Motivation

- Standard error model for an inertial sensor (IMU):

$$\hat{z} = \mathbf{M}_z \cdot (\mathbf{S}_z \cdot \mathbf{z} + \mathbf{b}_z) + \mathbf{w}_z$$

where:

$\mathbf{z}$:	Measurement
$\mathbf{M}$:	Misalignement matrix
$\mathbf{S}$:	Scale factors (const. and/or time-correlated)
$\mathbf{b}$:	Bias (const. and/or time-correlated)
$\mathbf{w}$:	Measurement noise (white)

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Errors in Inertial Sensors

Motivation

- Possible causes:
  - Non-orthogonalities of the sensor axes
  - Environmental conditions (e.g. temperature)
  - Electronics
  - Dynamics
  - ...
- Error types:
  - Deterministic (calibration models, physical models,...)
  - **Random error components** (stochastic processes,...)

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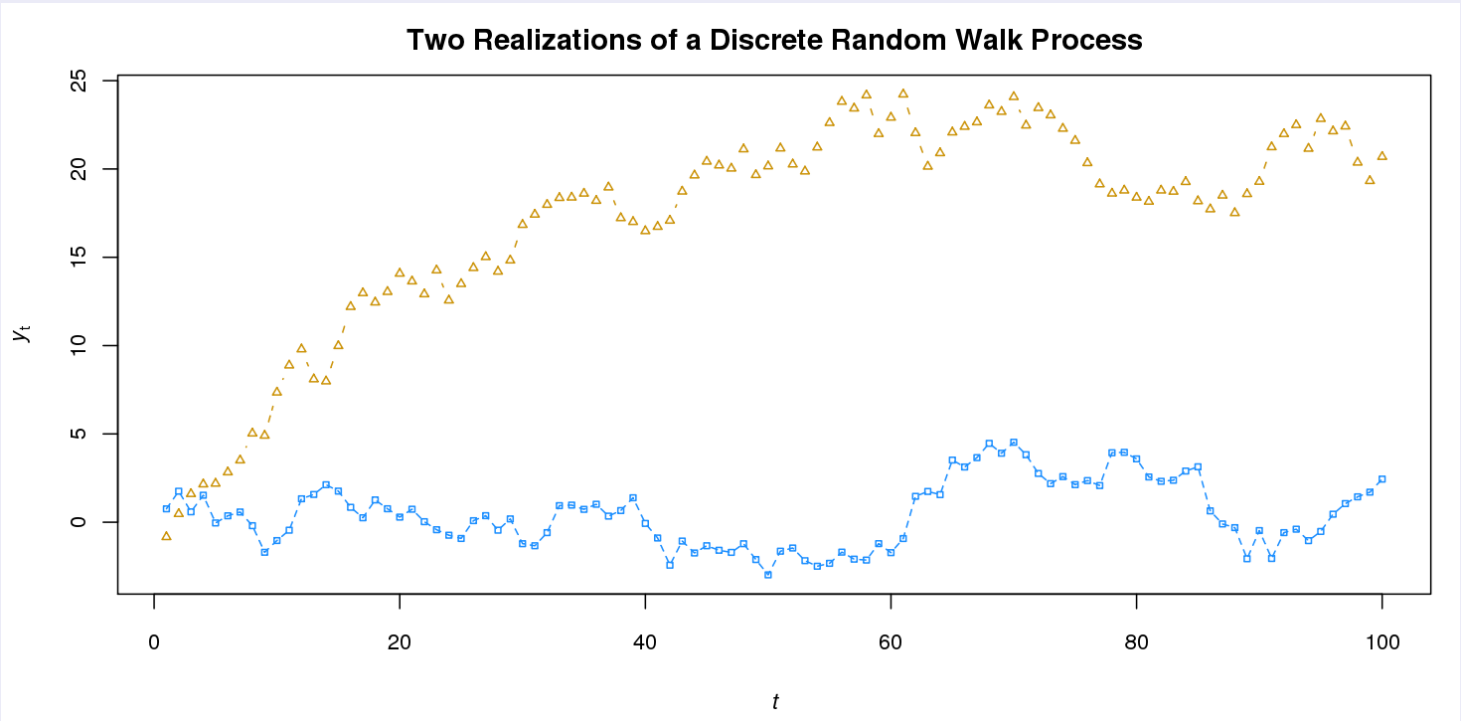
Here we Deal only with Random Errors...

Random errors are modeled through stochastic processes

Definition

A stochastic process is a collection of random variables associated with a deterministic parameter (time)



Importance of Error Modeling

Motivation

- Correct stochastic sensor error modeling implies:
  - Correct stochastic assumptions for estimation
  - Better navigation or post-processing performance

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Kalman Filter Tuning

Motivation

- A significant and time consuming process
- Sensor biases are modeled through augmented state variables
- Filter system and measurement models:

$$\begin{aligned} \boldsymbol{x}_k &= \boldsymbol{\Phi}_{k-1} \cdot \boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1} & p(\boldsymbol{w}_k) &\sim \mathcal{N}(0, \boldsymbol{Q}_k) \\ \boldsymbol{z}_k &= \boldsymbol{H}_k \cdot \boldsymbol{x}_k + \boldsymbol{v}_k & p(\boldsymbol{v}_k) &\sim \mathcal{N}(0, \boldsymbol{R}_k) \end{aligned}$$

Outline

1. Issues in Stochastic Process Parameter Estimation
   - Challenge
   - Classical Estimation Techniques
2. Goal of the Research
3. Generalized Method of Wavelet Moments
   - Theoretical Framework
   - GMWM Software
4. Application
   - Simulation Study
   - Experiments
5. Conclusion

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Lets Start with a small Example...

Simulation of a Stochastic Signal

Example

Lets simulate a signal $\{Y_t : t = 1, ..., 100000\}$ issued from a model F_θ composed of a:

- Gaussian white noise: $Y_{t,WN} = W_{WN},\ W_{WN} \sim \mathcal{N}(0, \sigma^2_{WN})$
- Rate ramp: $Y_{t,RR} = \omega \cdot t$
- First-order Gauss-Markov process:
  $Y_{t,GM} = e^{-\beta \Delta t} Y_{t-1} + W_{GM},\ W_{GM} \sim \mathcal{N}\left(0, \sigma^2_{GM}\left(1 - e^{2\beta\Delta t}\right)\right)$

Lets Start with a small Example...

Simulation of a Stochastic Signal

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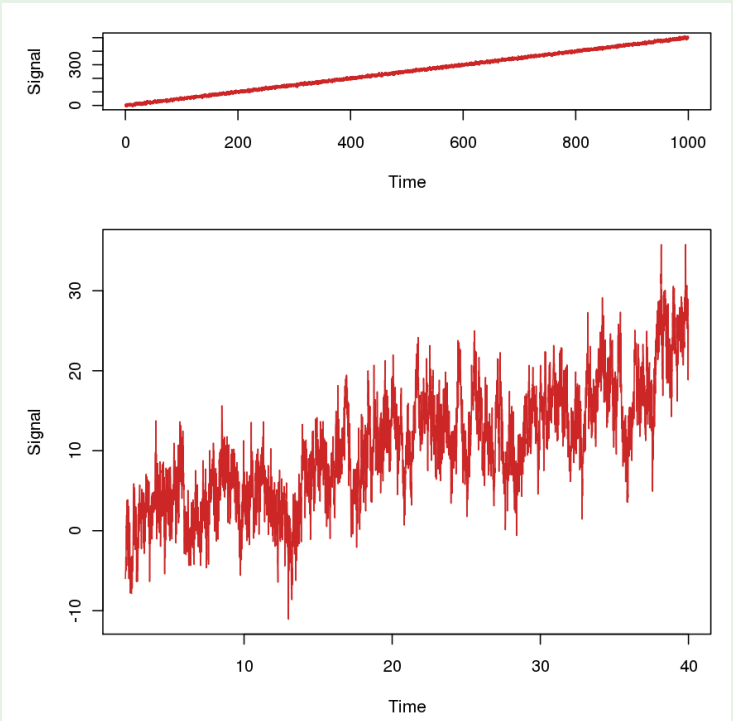

Lets Start with a small Example...

Simulation of a Stochastic Signal

Example

Question:

- How can we estimate $\boldsymbol{\theta} = \{\sigma^2_{WN}, \sigma^2_{GM}, \beta, \omega\}$ from $\{Y_t\}$?

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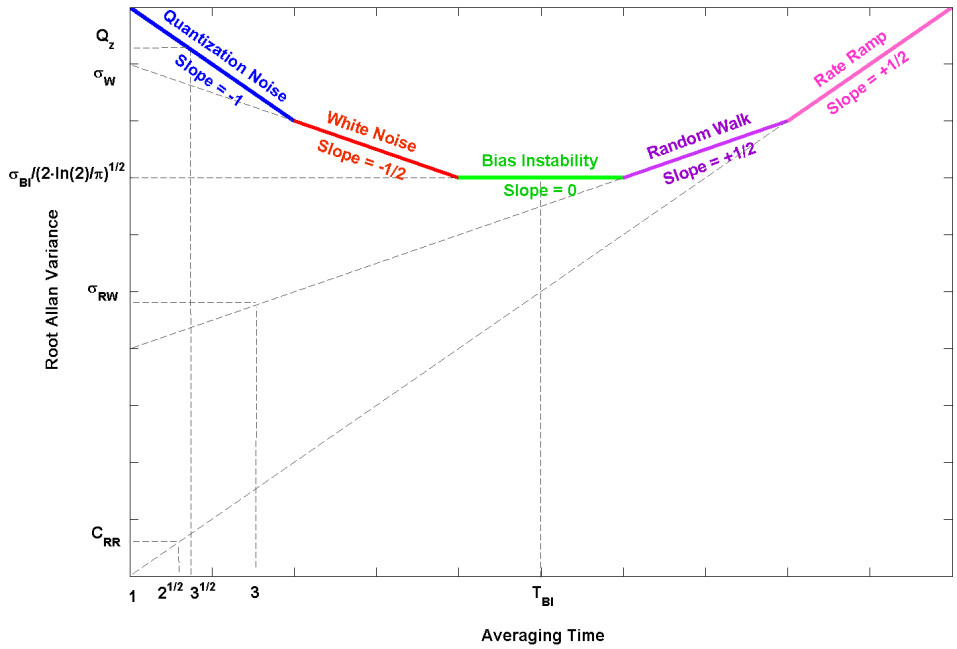
Classical Techniques

- Autocorrelation techniques
- Maximum likelihood based estimation (e.g. Expectation-Maximization)
- Allan Variance technique (visual identification):

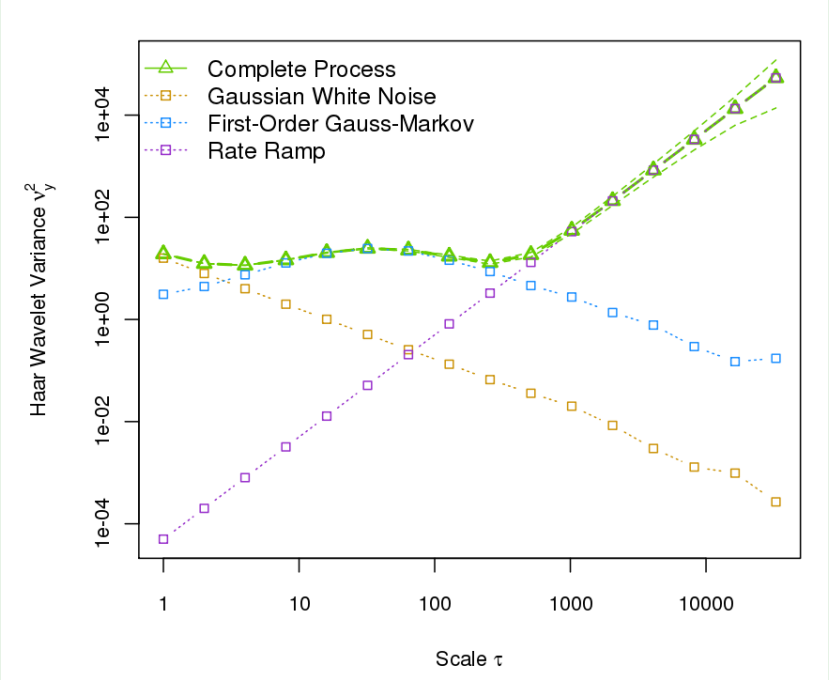


Back to our small Example...

Simulation of a Stochastic Signal

Example

Lets have a look on the Allan variance shape:



Challenge

- In stochastic process mixtures, estimating β and σ^2_{GM}, together with ω and σ^2_{WN} is tricky!
  - Researchers tried to use Allan Variance combined with denoising (limited success!)[3]
  - Adaptive Kalman filtering (system stability issues)[1]
  - Expectation-Maximization with Kalman Smoothing (possible convergence to local maxima in likelihood space) [2]
- **Hand tuning approach is most of the time used in practice!**
- Need of more sophisticated modeling techniques...

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Goal of the Research

- Given an error signal $\{Y_t : t \in \mathbb{Z}\}$ issued from a complex model $F_{\boldsymbol{\theta}}$ parameterized by $\boldsymbol{\theta}$:
  - How can we estimate θ if $F_{\boldsymbol{\theta}}$ is of complex spectral structure and $\{Y_t\}$ of stochastic nature?
- Development of a completely new framework for (stochastic) signal modeling

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General Principle of the Estimator

Merging two theories:

- Indirect Inference/Generalized Method of Moments (**statistical world**):
  - Instead of estimating $\boldsymbol{\theta}$ directly, we work on an auxiliary parameters linked with $\boldsymbol{\theta}$ through a binding function $\boldsymbol{\nu}(\cdot)$
- Wavelet powers are used as auxiliary parameters (**engineering world**)
  - MODWT wavelet coefficients for scale $\tau_j \equiv 2^{j-1}$ where $j = 1, 2, 3, \ldots$ are obtained by filtering $\{Y_t\}$ with $\{h_t\}$:

$$\overline{W}_{j,t} \equiv \sum_{l=0}^{L_j - 1} \widetilde{h}_{j,t} Y_{t-l}, \; t = \ldots -1, 0, 1, \ldots$$

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Wavelet Filters as Spectrum Analyzer

Wavelet Power: Definition

Definition

Wavelet power:

$$\nu_y^2(\tau_j) \equiv \mathrm{var}\{\overline{W}_{j,t}\}$$

Wavelet Filters as Spectrum Analyzer

Wavelet Power : Variance Decomposition & Power Spectrum Substitute

- Wavelet power decomposes variance over scales τ_j

$$\sum_{j=1}^{\infty} \nu_y^2(\tau_j) = \sigma_y^2 \equiv \mathrm{var}_{F_\theta}\{y_t\}$$

- $\{\nu_y^2(\tau_j)\}$ provides an octave-band estimate of the power spectrum Φ_{yy}

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Wavelet Filters as Spectrum Analyzer

Wavelet Power : Unbiased Estimator

- An **unbiased** estimator of $\nu_y^2(\tau_j)$ for $\{Y_t : t = 1, ..., N\}$ is:

$$\hat{\nu}_y^2(\tau_j) = \frac{1}{M(T_j)} \sum_{t \in T_j} \overline{W}_{j,t}^2$$

where T_j is the set of time indices for which $\{\overline{W}_{j,t}^2\}$ are not affected by boundary effect in circular filtering

- We demonstrated that the asymptotic distribution of $\hat{\nu}^2$ is given by:

$$\sqrt{T}(\hat{\nu}^2 - \mathbb{E}[\hat{\nu}^2]) \longmapsto \mathcal{N}(0, \mathbf{V}_{\hat{\nu}^2}), \ T \to \infty$$

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Principle of the Estimator

Generalized Least-Squares Principle (GLS) using Wavelet Power

- Indirect inference means solving the following optimization problem:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}\in\boldsymbol{\Theta}}{\operatorname{argmin}} \left(\hat{\boldsymbol{\nu}}^2 - \hat{\boldsymbol{\nu}}^2(\boldsymbol{\theta})\right)^T \boldsymbol{\Omega} \left(\hat{\boldsymbol{\nu}}^2 - \hat{\boldsymbol{\nu}}^2(\boldsymbol{\theta})\right)$$

- There exist a mapping (which can be inverted):

$$\theta \mapsto \nu^2(\theta)$$

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Principle of the Estimator

Generalized Least-Squares Principle (GLS) using Wavelet Power

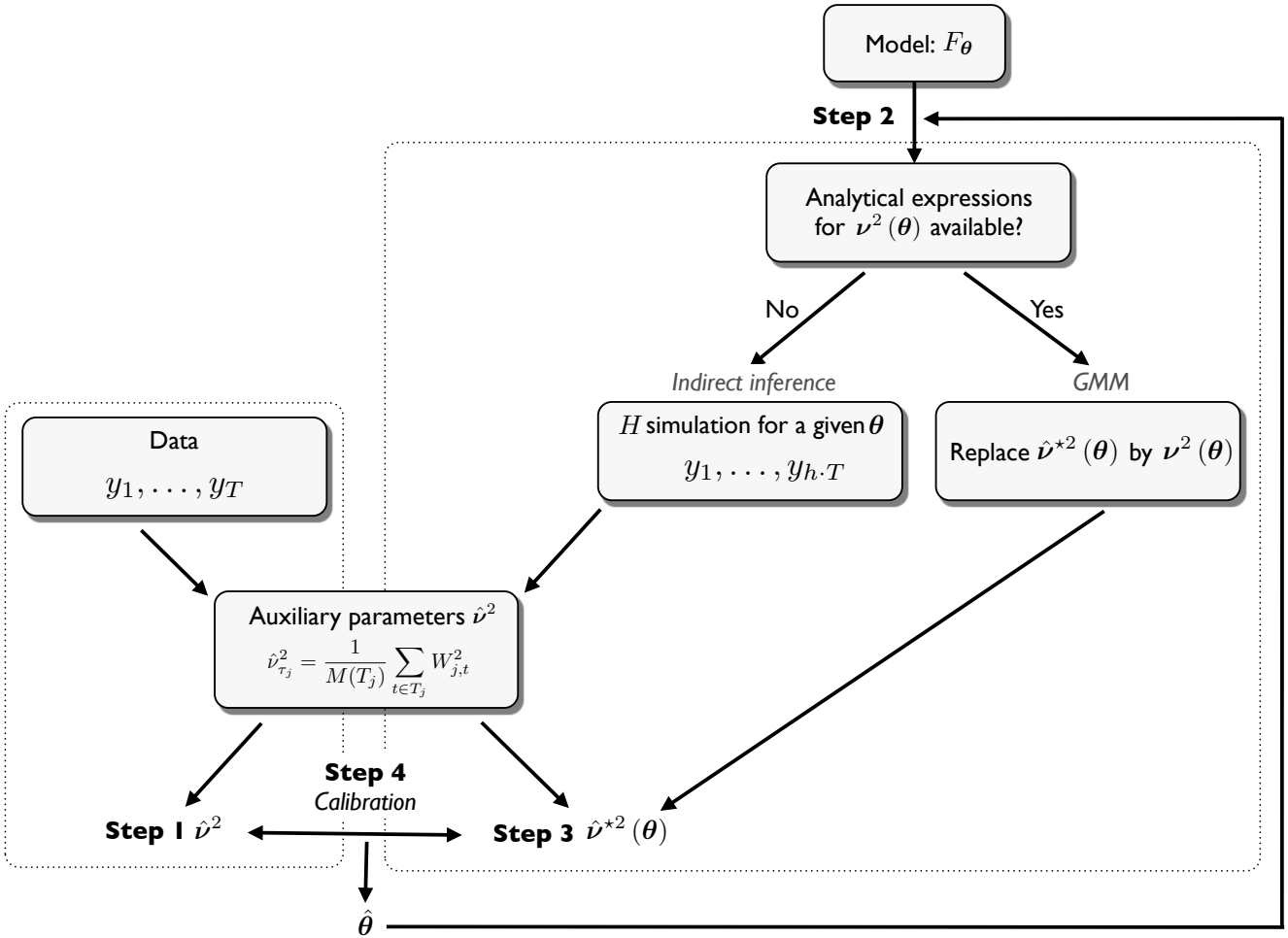
- When no analytical expression for $\boldsymbol{\nu}^2(\boldsymbol{\theta})$ is available, we can solve:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}\in\boldsymbol{\Theta}}{\operatorname{argmin}} \left(\hat{\boldsymbol{\nu}}^2 - \hat{\boldsymbol{\nu}}^2_*(\boldsymbol{\theta})\right)^T \boldsymbol{\Omega} \left(\hat{\boldsymbol{\nu}}^2 - \hat{\boldsymbol{\nu}}^2_*(\boldsymbol{\theta})\right)$$

$\hat{\boldsymbol{\nu}}^2_*(\boldsymbol{\theta})$ are computed on **synthetic signals** $\{Y^*_t(\boldsymbol{\theta}),\ t = 1, ..., R \cdot T\}$, $R \geq 1$ driven from $F_{\boldsymbol{\theta}}$

Principle of the Estimator

Generalized Least-Squares Principle (GLS) using Wavelet Power

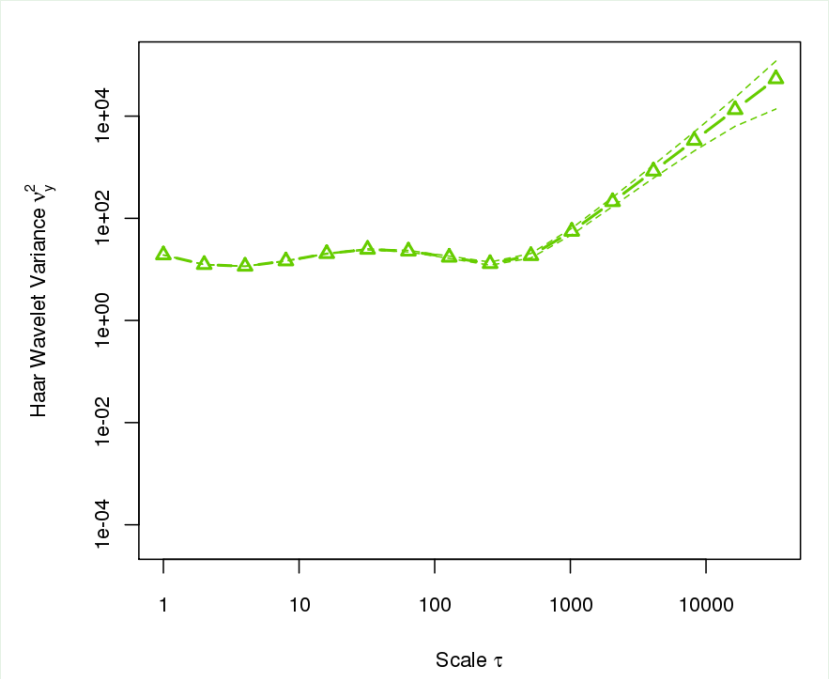


Principle of the Estimator

Lets return to our Simulation Study...

Example

This is the $\hat{\nu}_y^2(\theta)$ sequence:



Principle of the Estimator

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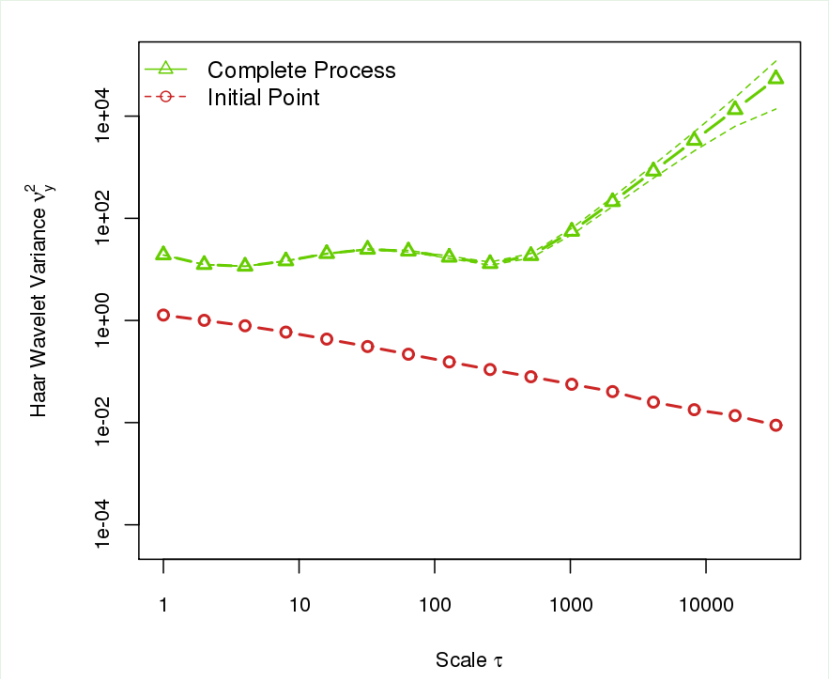
Example

Start at an arbitrary point $\hat{\theta}_0$ and compute $\hat{\nu}_*^2(\theta)$:

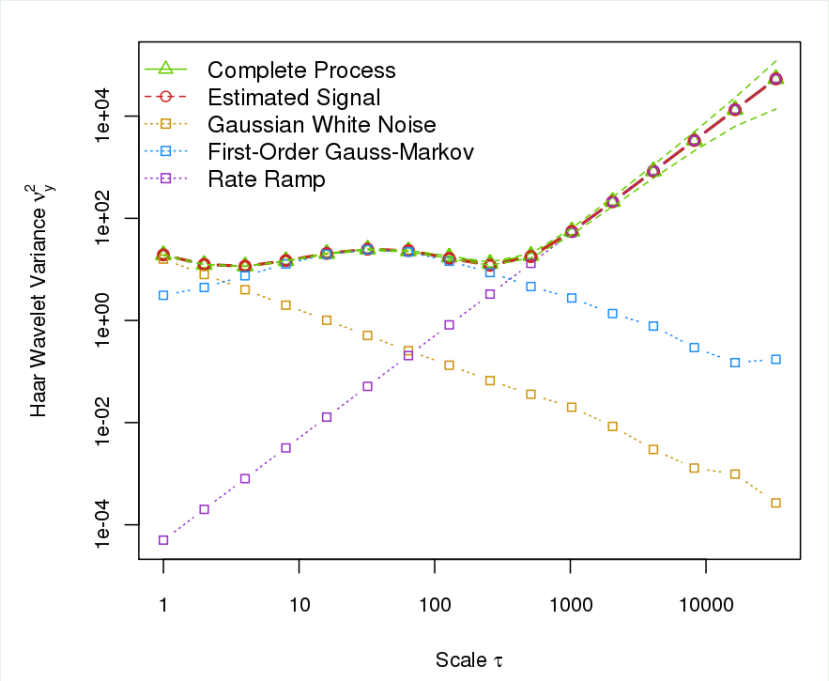


Principle of the Estimator

Lets return to our Simulation Study...

Example

Perform optimization to find $\hat{\theta}$:



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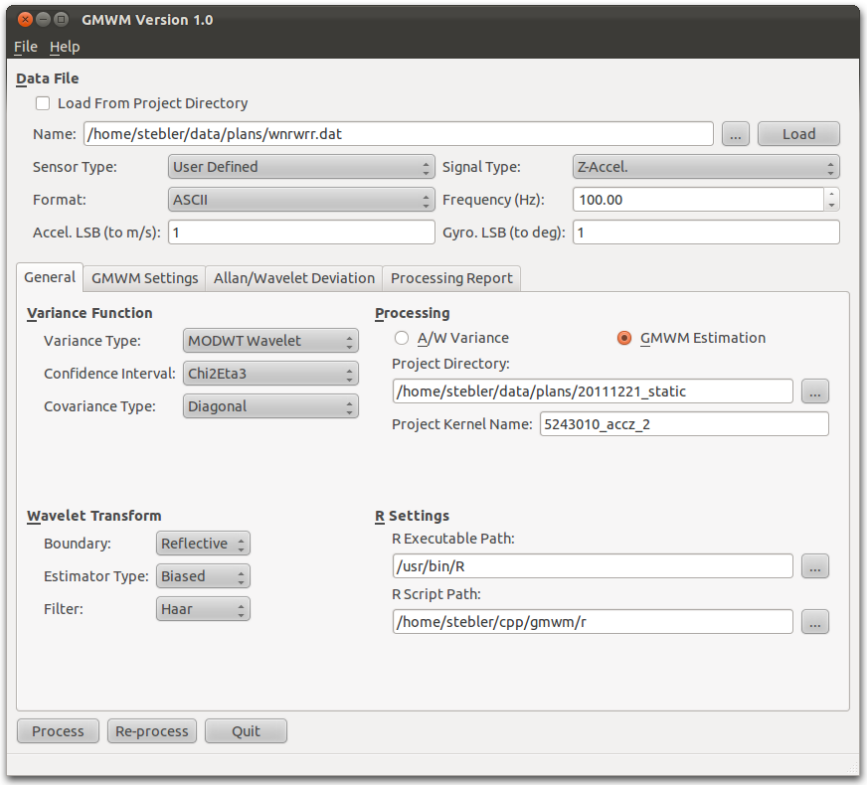
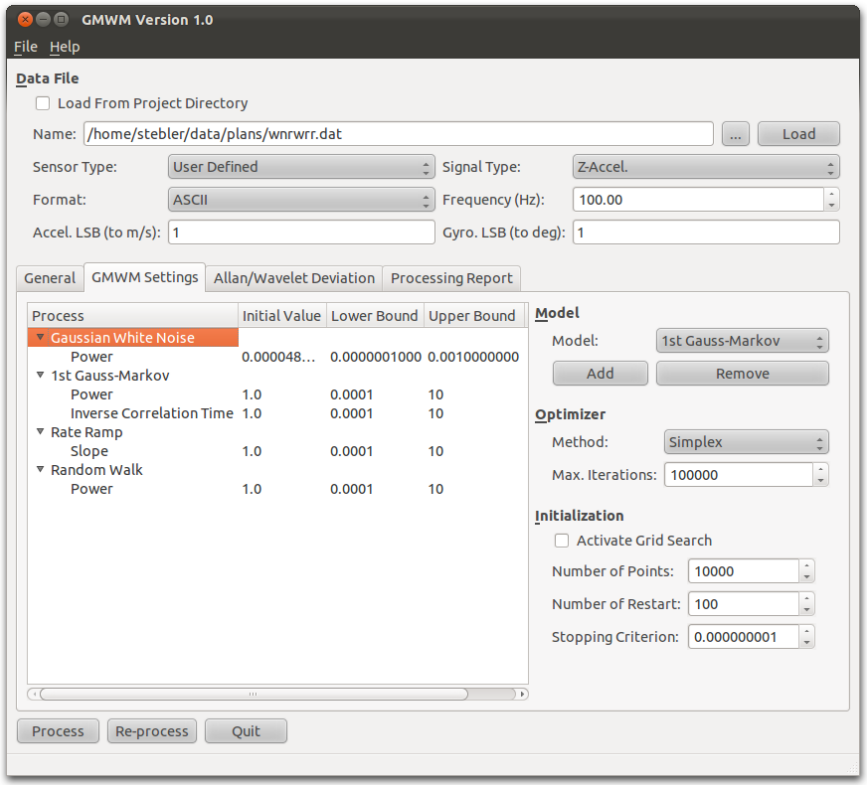
GMWM 1.0

C++ based software





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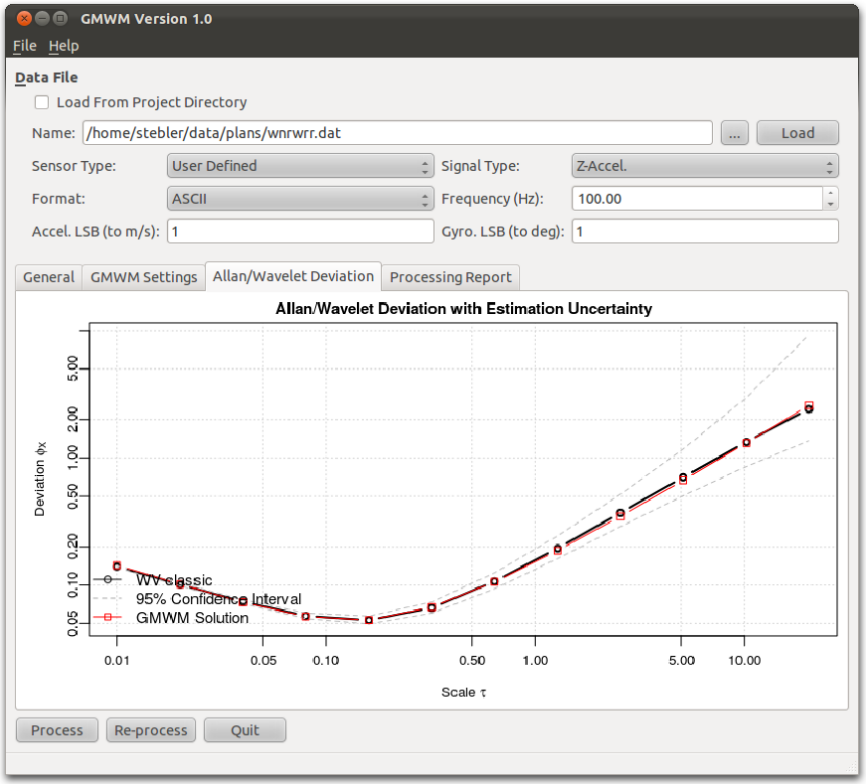

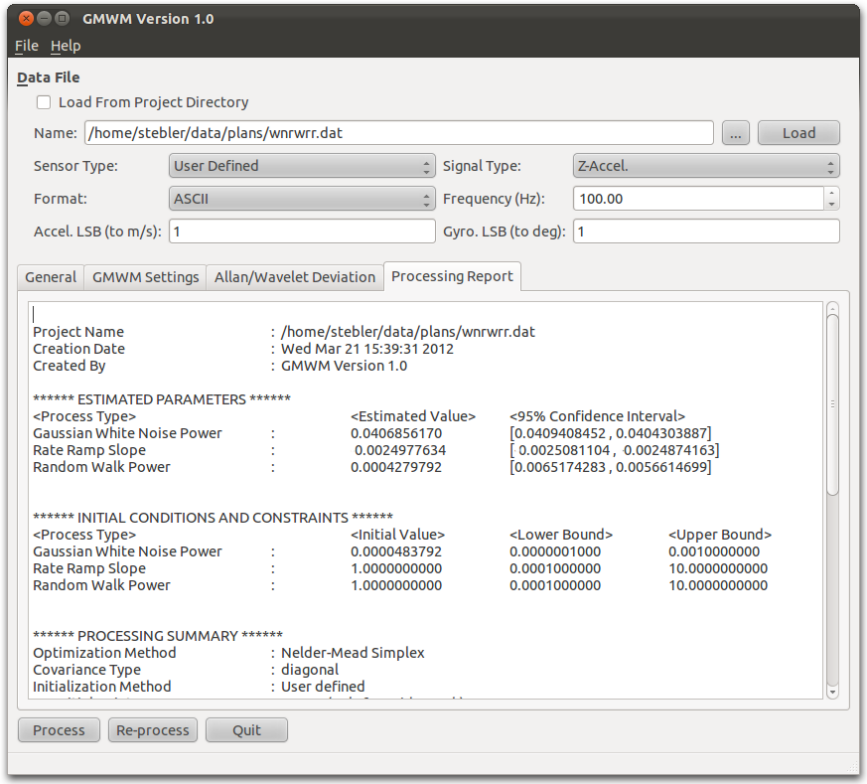

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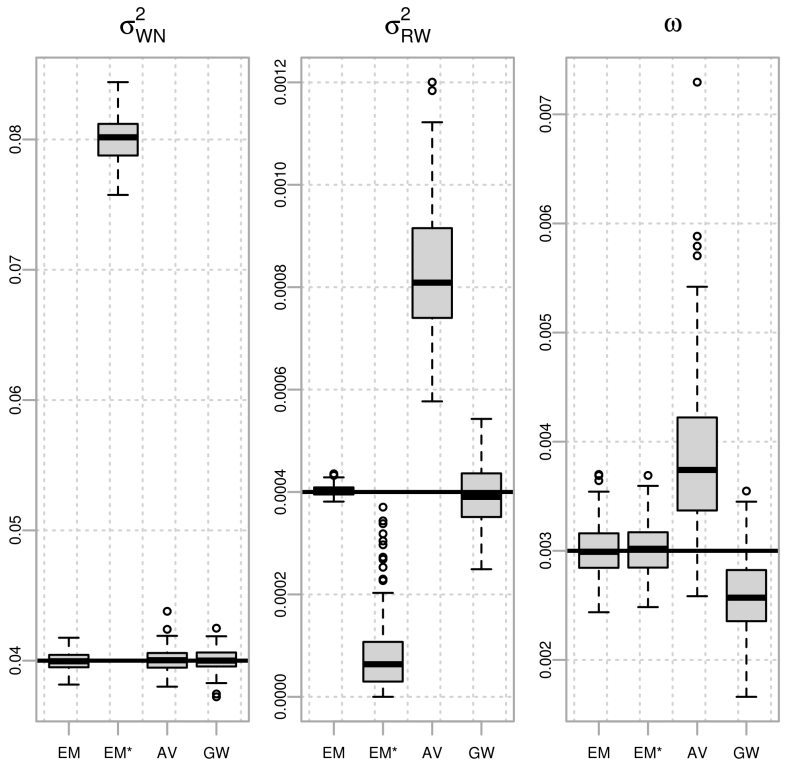
White Noise + Random Walk + Drift (Rate Ramp)

The model F_θ is:

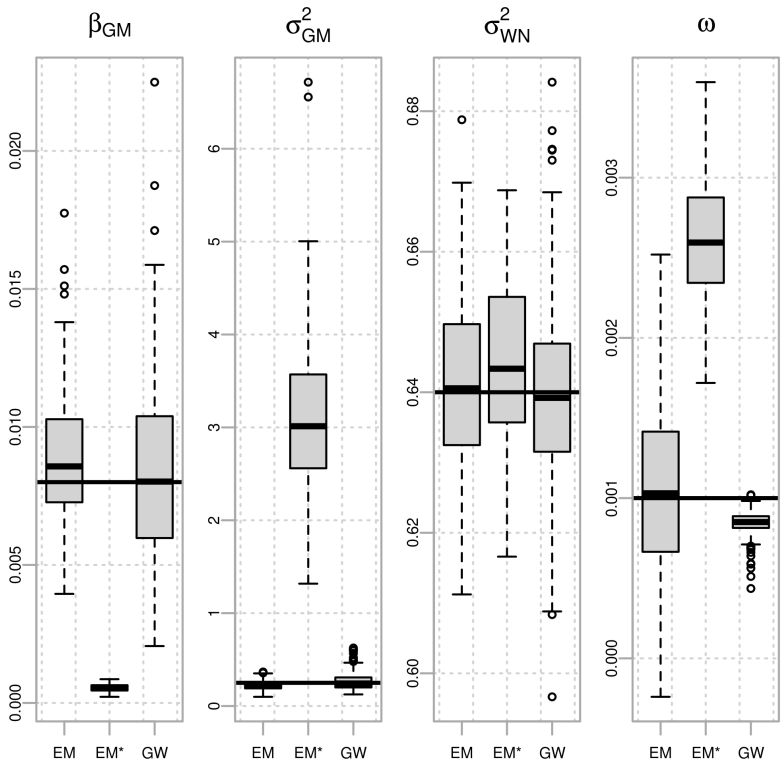
$$Y_t = Y_{t,WN} + Y_{t,RW} + Y_{t,RR}$$



White Noise + Gauss-Markov (1st order) + Drift

The model F_θ is:

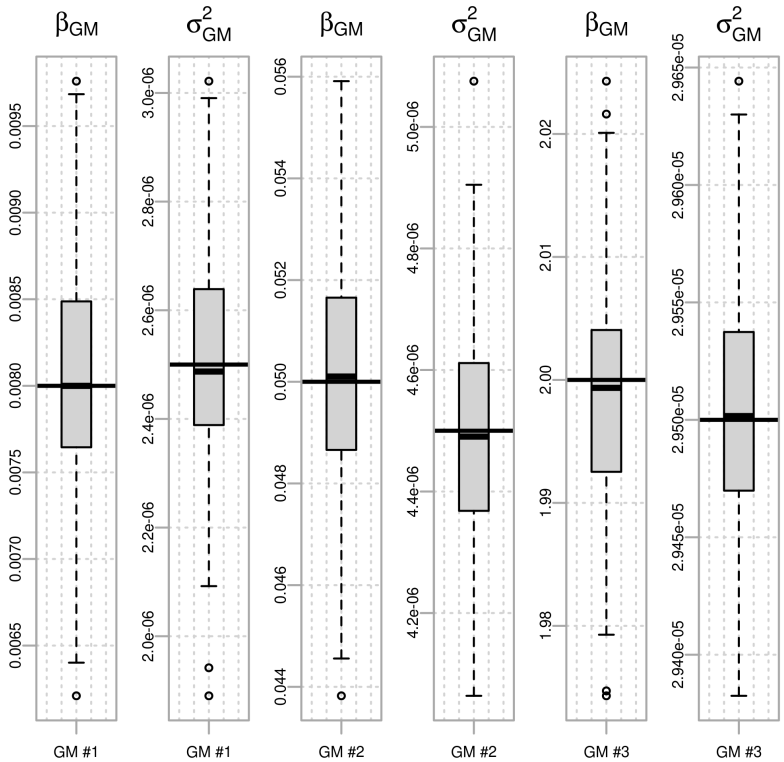
$$Y_t = Y_{t,WN} + Y_{t,GM} + Y_{t,RR}$$



Sum of Three Gauss-Markov Processes

The model F_θ is:

$$Y_t = \sum_{k=1}^{3} Y_{t,GM}^{(k)}$$



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Experiment Setup

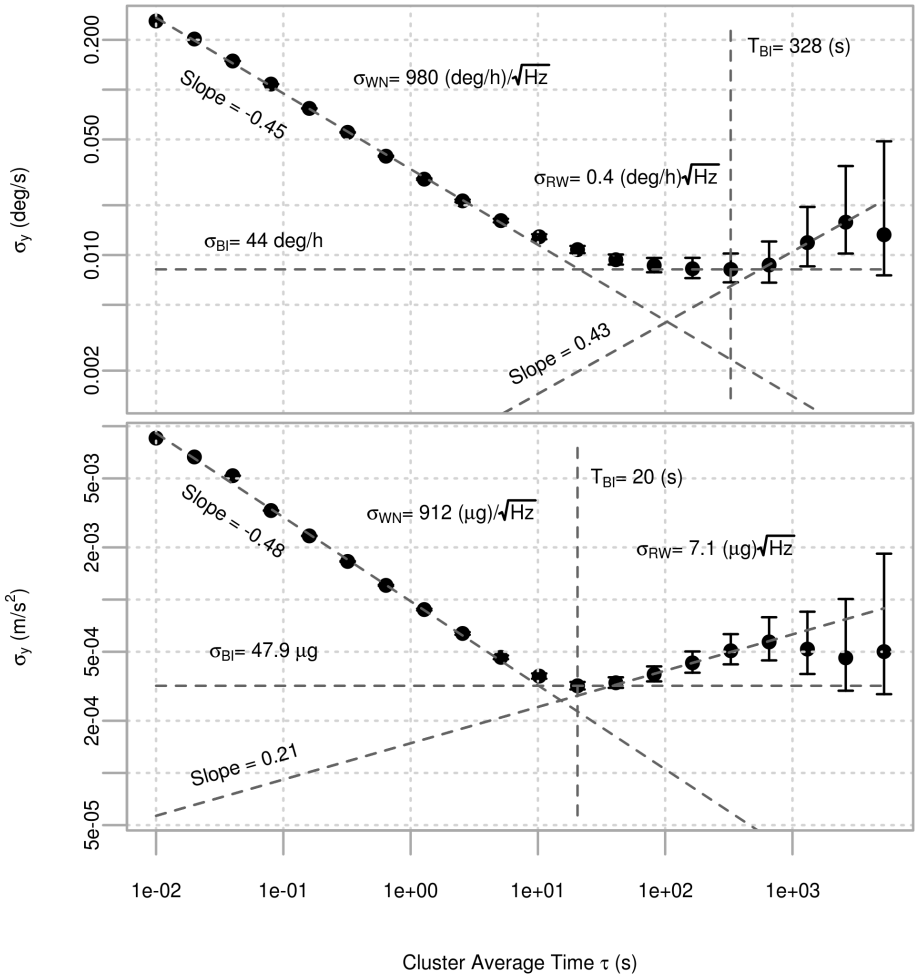
- Static data collected during 4.5 hours @100Hz
- Constant temperature condition
- XSens MTi-G IMU

Experiment Setup

Sensor calibration using the following approaches:

- Allan Variance (AV)
- KF-(Self)-tuning approach
- GMWM approach

Allan Variance (Deviation) Analysis



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KF-(Self)-Tuning Approach

Procedure:

- AV parameters used as initial approximation
- Analysis of position drift during articial GNSS outages
- L1/L2 carrier-phased DGPS reference solution

The model F_θ is:

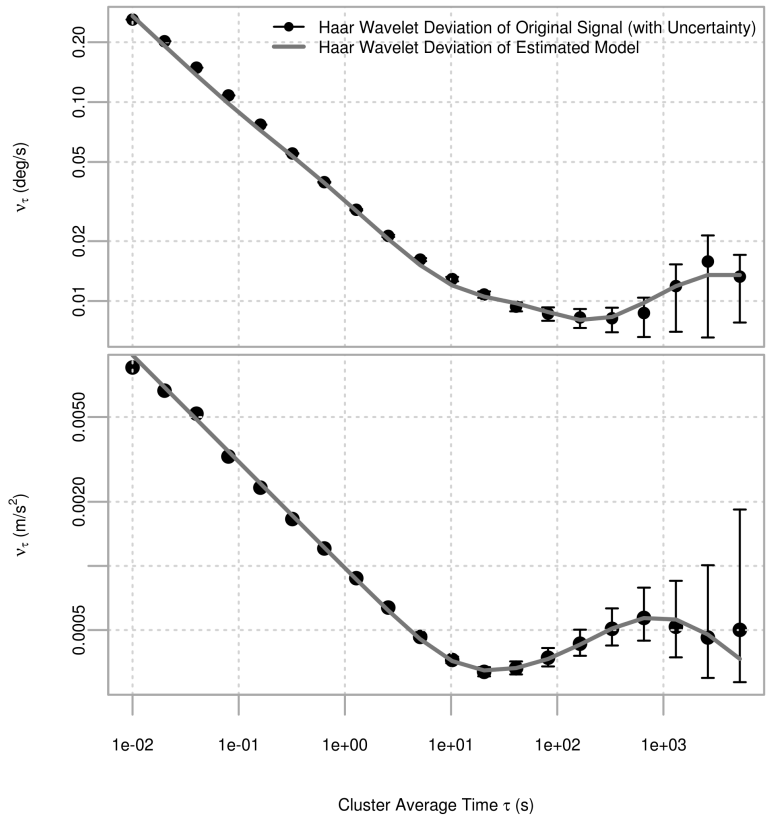
$$Y_t = Y_{t,WN} + Y_{t,GM}$$

GMWM Approach

- We considered the following model $F_{\boldsymbol{\theta}}$:

$$Y_t = Y_{t,WN} + \sum_{k=1}^{g} Y_{t,GM}^{(k)}, \; g = 3$$

- Estimation results:



Validation

- Validating and comparing the quality of models is a non-trivial task:
  - True model F_{θ} is unknown
  - Calibration on signals acquired in static conditions

Validation

- Procedure:
  1. INS/GNSS reference solution (L1/L2 DGPS + tactical/navigation grade IMU)
  2. Emulation of synthetic IMU and GNSS signals along the reference
  3. Addition of real static noise signal on synthetic signals
  4. Introduction of artifical GNSS gaps
  5. Processing using closed-loop EKF implementing the models
  6. Quality juged by analyzing the navigation error and EKF-predicted accuracy during inertial coasting mode

Validation

Reference: Helicopter trajectory (Litton LN200 tactical-grade IMU @400Hz + Javad Legacy L1/L2 GNSS Rx @10Hz)

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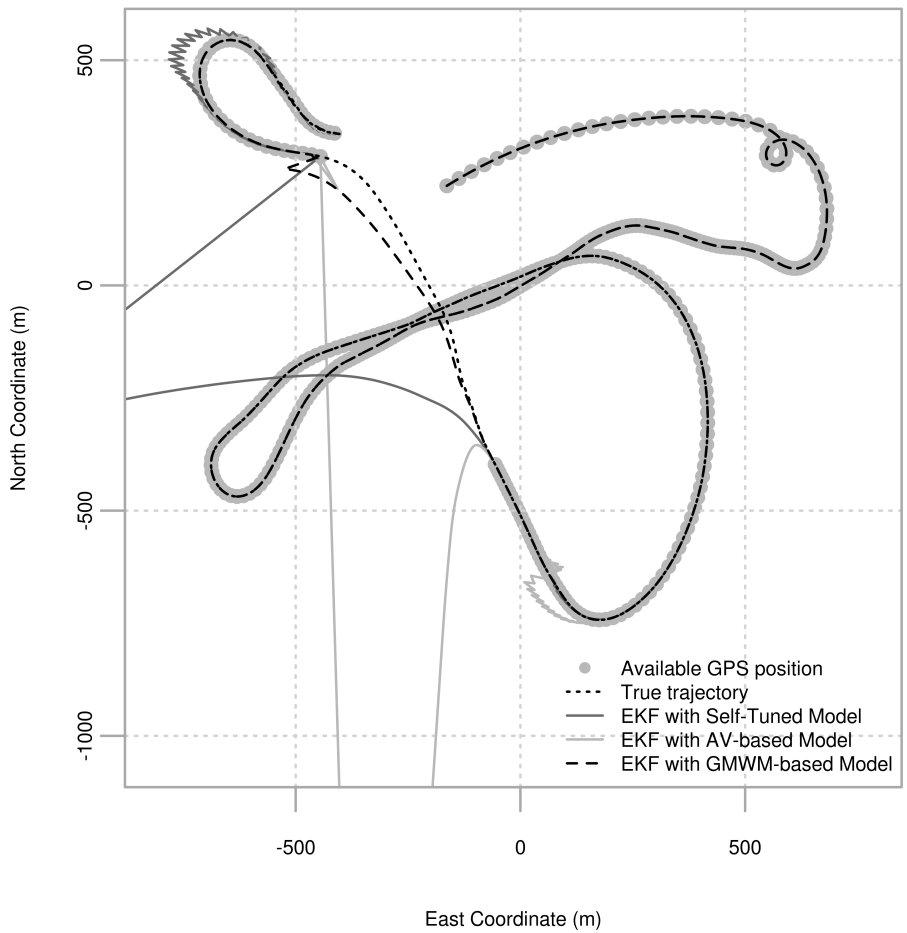

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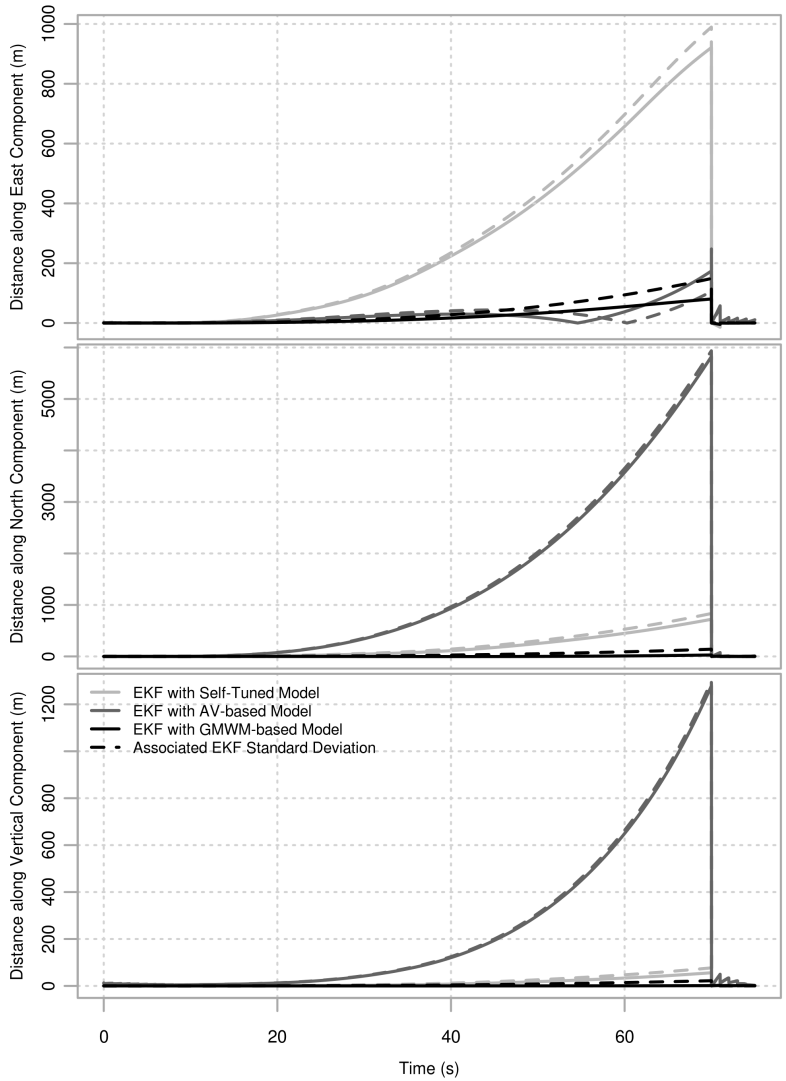
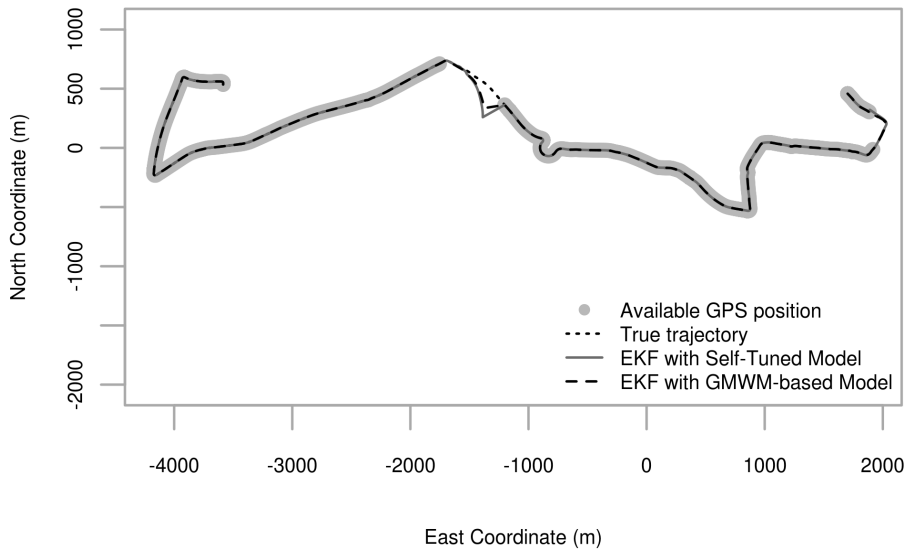


Validation

Reference: Car trajectory (IXSEA Airins navigation-grade IMU @100Hz + Javad Delta L1/L2/Glo GNSS Rx @10Hz)

Validation

Reference: Car trajectory (IXSEA Airins navigation-grade IMU @100Hz + Javad Delta L1/L2/Glo GNSS Rx @10Hz)



Validation

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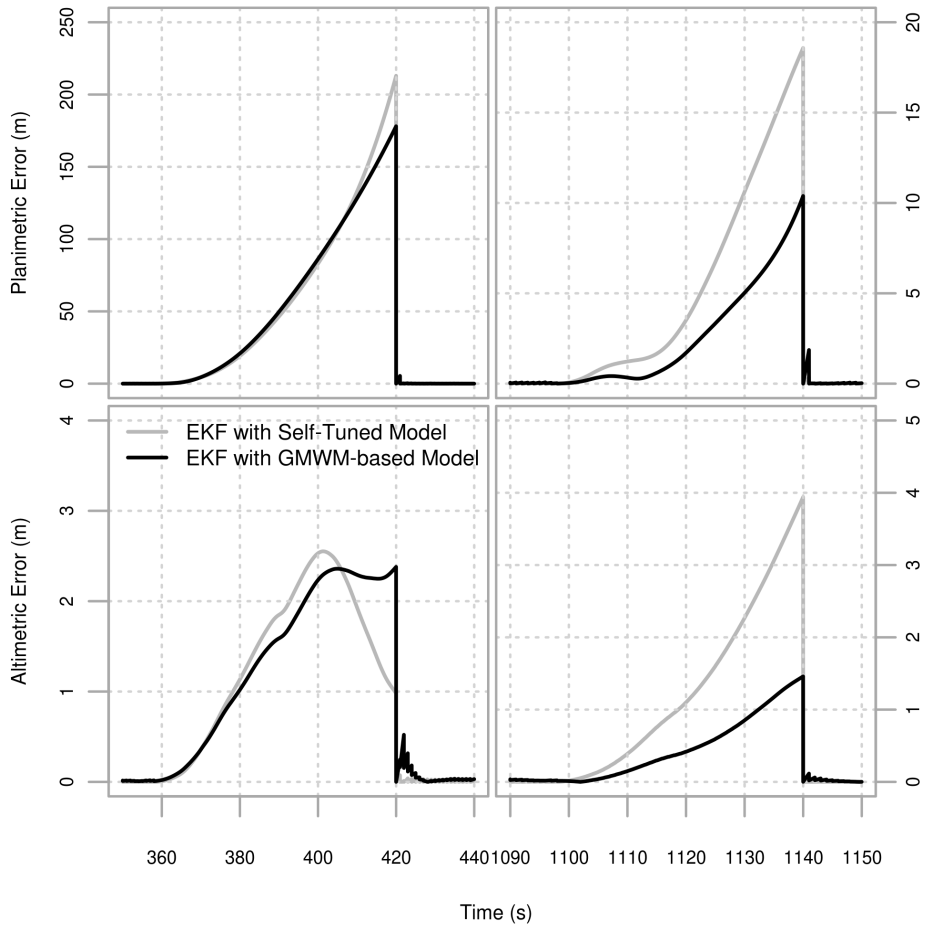

Conclusion

- The GMWM estimator...:
  - ...is able to model signals of complex spectral structure (classical method fail!)
  - ...can provide confidence intervals (not trivial, not shown here)
  - ...has a completely new theoretical framework
  - ...can be used for any type of stochastic signal
- The importance of correct sensor calibration in EKF was demonstrated through navigation accuracy analysis in inertial coasting mode

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